

Performance Comparison of FFT and DWT based OFDM and Selection of Mother Wavelet for OFDM

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Abstract — In this paper a comparative study on DWT-OFDM and FFT-OFDM systems is presented. The DWT-OFDM has to satisfy the orthonormal basis and the perfect reconstruction properties to be considered for OFDM. Different wavelet families have been used and BER performance is compared with the conventional FFT-OFDM system for AWGN. The results show that the DWT-OFDM system operates at its optimum performance with different wavelets. Results also show that DWT-OFDM is superior as compared to FFT-OFDM with regards to the bit error rate (BER) performance in AWGN channel.

Keywords — *Discrete Wavelet Transform, Discrete Wavelet Based OFDM (DWT-OFDM), Fourier-based OFDM (FFT-OFDM), Orthogonal Frequency Division Multiplexing (OFDM)*

I. INTRODUCTION

The desire for faster wireless technologies and the increase in multimedia applications is the principal driving force behind OFDM's increased popularity. Orthogonal Frequency Division Multiplexing (OFDM) is a modulation scheme used to transmit digital data efficiently in multipath fading environment. It is a multi-carrier system with simultaneous transmission of data on closely spaced orthogonal sub-carriers. Conventional OFDM system is deployed with IFFT (Inverse Fourier Transform) to generate orthogonal sub-carriers. A cyclic prefix is added to every symbol to combat the delay spread introduced by multipath channel and the length of cyclic prefix must be at least the same size as the expected channel delay spread. Addition of cyclic prefix minimizes inter-symbol-interference (ISI) [1].

With the rapid growth of digital communication in recent years, the need for high-speed data transmission has been increased. The wireless industry faces the problem of providing the technology that be able to support a variety of services ranging from voice communication with a bit rate of a few kbps to wireless multimedia in with a bit rate up to 2 Mbps. Many systems have been proposed and OFDM system has gained much attention for different reasons. Although OFDM was first proposed in the 1960s, only in recent years, it has been recognized as an outstanding method for high-speed data communication system where

its implementation relies on very high-speed digital signal processing. Since OFDM is carried out in the digital domain, OFDM method is flexible for the design process and enough fast in terms of time to put it in the market [2].

OFDM is a multicarrier modulation technique. OFDM provides high bandwidth efficiency because the carriers are orthogonal to each other and multiple carriers share the data among themselves. The main advantage of this transmission technique is their robustness to channel fading in wireless communication environment. The main objective of this project is to implement and test an OFDM transmitter and receiver using a FPGA at baseband with Discrete Wavelet Transform. As a first step we have designed a simulation model using MATLAB to check the performance of the paper design [3].

Orthogonal Frequency Division Multiplexing (OFDM) is a multi-carrier modulation technique which divides the available spectrum into many carriers. OFDM uses the spectrum efficiently compared to FDMA by spacing the channels much closer together and making all carriers orthogonal to one another to prevent interference between the closely spaced carriers. The main advantage of OFDM is their robustness to channel fading in wireless environment. Commercial applications of OFDM are ADSL, IEEE802.11a/HiperLAN2, WiMAX, Digital Broadcasting (DVB-T) [4].

This paper is organized as follows. Section II explains the basic FFT-OFDM system with equations. Section III gives the need of different wavelets in OFDM system design. Section IV illustrates the implementation of Discrete Wavelet Transform in OFDM system with section V .Analytical model of OFDM system and simulation parameters are explained in section VI and section VII respectively. Finally, we concluded this paper by appropriate results and future implementation in section VIII and section IX respectively.

II. FOURIER BASED OFDM (FFT-OFDM)

Maintaining orthogonality between the sub carriers is the key factor in generating OFDM signal. The signal is generated by firstly choosing the spectrum required, based on the input data, and modulation scheme used. Each

carrier to be produced is assigned some data to transmit. The required amplitude and phase of the carrier is then calculated based on the modulation scheme (typically BPSK, differential BPSK, QPSK, or QAM).

An efficient implementation of OFDM transmitter and receiver can be built with the inverse fast Fourier transform (IFFT) and the FFT, respectively. The structure of generic baseband OFDM transmitter and receiver is shown in fig.1.

The following figure shows the basic block diagram of OFDM system. In this system we have to give random bits as an input. By using BPSK modulation technique we have to generate symbols. To get number of subcarriers to inculcate we have to provide appropriate IFFT order. Cyclic prefix is added at each symbol length to avoid inter carrier interference (ICI) and inter symbol interference (ISI).

$$X_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_n e^{2\pi jkn/N} \tag{1}$$

An OFDM transceiver system is shown in Fig. 1. The inverse and forward transform blocks are concerned in more attentions since they can be FFT-based or DWT-based OFDM. The data generator producing random binary form. It is first being processed by a constellation mapping. BPSK modulator is used for this work to map the raw binary data to appropriate BPSK symbols. These symbols are then input into the IFFT block. This involves taking N parallel streams of symbols (N being the number of subcarriers used in the transmission of the data) and performing an IFFT operation on this parallel stream. The output in discrete time domain is as follows:

$$x_n = \frac{1}{\sqrt{N}} \sum_{K=0}^{N-1} X_K e^{-2\pi jkn/N} \tag{2}$$

To maintain the orthogonality during channel transmission cyclic prefix is added to OFDM frame which must be longer than the channel impulse response. Channel performance can be estimated and hence equalized by inserting pilot subcarriers at predefined subcarrier intervals. The sub-channels are detected by computing DFT of the scaled samples of an OFDM symbols at the receiver.

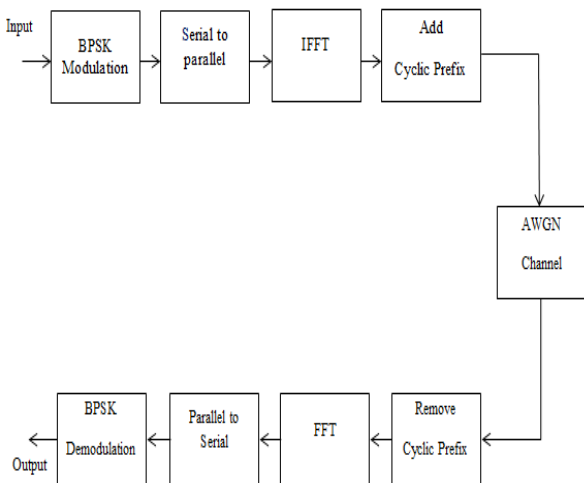


Fig.1.FFT based OFDM System

III. MOTIVATION FOR USING WAVELETS.

There are several advantages of using wavelets for wireless communication systems. The few desirable features of wavelets are the following:

- Wavelet transform can create subcarriers of different bandwidth and symbol length.
- The ability of wavelets to arrange the time-frequency tiling in a manner that minimizes the channel disturbances minimizes the effect of noise and interference on the signal.
- Wavelets give a new dimension, signal diversity which could be exploited in a cellular communication system, where adjacent cells can be designated different wavelets in order to minimize inter-cell interference.
- Wavelet-based algorithms have long been used for data compression. By compressing the data, a reduced volume of data is transmitted so that the communication power needed for transmission is reduced.
- Mitigation of Interference ISI and ICI: ISI (Inter symbol interference) and ICI (Inter Carrier Interference) are influenced by shape of the basic pulse. Time dispersion contributes interference due to multipath effect and frequency dispersion due to non-linear effects of radio channel. In wireless scenario the channel effects cannot be controlled. But the pulse shape can be carefully designed to have minimum distortion for a given delay spread. The wavelet transform allows more flexibility in the design of the pulse shape. Many researchers proved that the wavelet based multi-carrier schemes are superior in suppressing ICI and ISI as compared to the traditional Fourier based systems.

IV. WAVELETS FOR MULTICARRIER MODULATION

Wavelets are small waveforms with a set oscillatory structure that is non-zero for a limited period of time (or space) with additional mathematical properties. The wavelet filters are characterized with wavelet bases. These signal transmission techniques allow for significant increase in wireless capacity without increase in bandwidth. FFT based OFDM is a Multi-Carrier Modulation (MCM) scheme where the sub carriers are orthogonal. The rectangular window used in its implementation creates high side lobes. Whereas, DWT-OFDM has longer basis functions and hence can offer higher degree of side lobe suppression. The transmitted signal $x[k]$ is composed of successive digital modulated symbols, each of which is constructed as the sum of M waveforms $\Phi(k)$ individually amplitude modulated. It can be expressed in the discrete domain as:

$$x[k] = \sum_s \sum_{m=0}^{M-1} (a_j, \Phi_m[k - sM]) \tag{3}$$

In DWT-OFDM, the discrete functions $\Phi(k)$ are the complex exponential basis functions. The transmitted

symbol is built by performing inverse DWT while the forward DWT is used to retrieve the data symbol. The carrier waveforms are obtained by iteratively filtering the signal into high and low frequency components. The relationship between the number of iterations J and the number of carrier waveforms M is given by $M = 2^J$.

There are few wavelets in literature that are recognized by most authors. Each wavelet has some distinguishing characteristics that make it more suitable for one application than other. While designing a system the careful consideration of different wavelet properties should be made with respect to the system requirements. The selection of wavelets is generally made on the following wavelet properties:

- **Compact Support:** Compact support is defined by the length (L) of the filter. Shorter filters have decreased computational complexity.
- **Orthogonality:** It is one of the most vital wavelet properties as it ensures perfect reconstruction. For communication purposes we absolutely require orthogonal wavelets.
- **Symmetry:** Symmetrical wavelets have a feature that transform of the mirror of an image is the same to the mirror of the wavelet transform. None of the orthogonal wavelets except Haar wavelet is symmetric.
- **K-Regularity/Vanishing Moments:** K -regularity is also an important measure for wavelets because it helps to reduce non-zero coefficients in the high-pass sub-bands and it is one of the easiest ways to determine if a scaling function is fractal or not.

V. DISCRETE WAVELET BASED OFDM SYSTEM (DWT-OFDM)

This section discusses the alternative way to implement OFDM using DWT. In Wavelet based OFDM (DWT-OFDM), the time-windowed complex exponentials are replaced by wavelet "carriers", at different scales (j) and positions on the time axis (k). These functions are generated by the translation and dilation of a unique function, called "wavelets mother" and denoted by $\psi(t)$:

$$\Psi_{j,k}(t) = 2^{-j/2} \Psi(2^{-j}t - k) \tag{4}$$

The orthogonality of these carriers relies on time location (k) and scale index (j). Wavelet carriers exhibit better time-frequency localization than complex exponentials while DWT-OFDM implementation complexity is comparable to that of FFT- OFDM. The key point 'orthogonality' is achieved by generating members of a wavelet family, according to equation (4)

$$\langle \Psi_{j,k}(t), \Psi_{m,n}(t) \rangle = \begin{cases} 1, & j = m \ \& \ k = n \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

These functions have orthonormal basis of $L^2(R)$, if infinite number of scales $j \in Z_j$ are considered. To obtain finite number of scales, scaling function $\Phi(t)$ is used. DWT-OFDM symbol now can be considered as weighted sum of wavelet and scale carriers, as expressed in equation (6). This is close to the Inverse Wavelet Transform (IDWT).

$$s(t) = \sum_{j \leq J} \sum_k w_{j,k}(t) \cdot \Psi_{j,k}(t) + \sum_k a_{J,k} \cdot \Phi_{J,k}(t) \tag{6}$$

Fig.2. indicates the basic block diagram DWT-OFDM where we do not have to add cyclic prefix at the transmitter section because of wavelet properties as discussed in section. The data symbols are seen by IDWT modulator as sequence of wavelet $w_{j,k}$ and approximation coefficients $a_{j,k}$.

According to Equation (4) J is the scale with poorest time resolution and best frequency localization of the carriers. For computing IDWT, Mallet's algorithm based on filter bank is used instead of Equation (6). At the output of the filter discrete version of DWT-OFDM symbol is obtained, with impulse response of filters (low-pass and high-pass) decided by the mother wavelet.

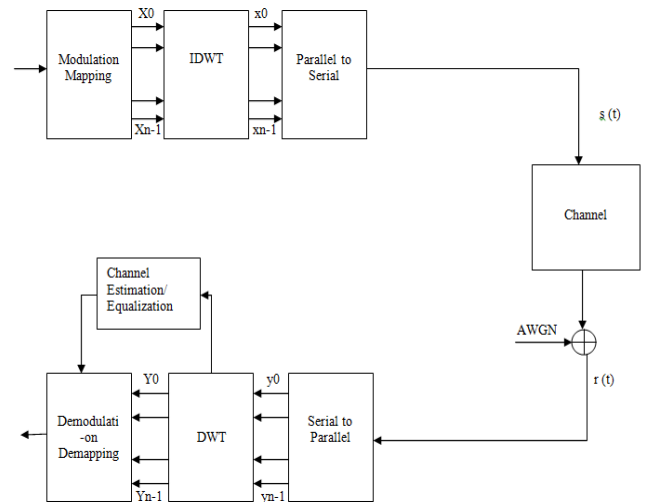


Fig.2.DWT based OFDM System

VI. ANALYTICAL MODEL OF OFDM SYSTEM

The analytical BER expressions for BPSK signalling for AWGN channel and FFT-OFDM system is given by

$$P_e = \frac{2(M-1)}{M \log_2 M} Q \left(\sqrt{\frac{6E_b}{N_0}} \cdot \sqrt{\frac{\log_2 M}{M^2 - 1}} \right) \tag{7}$$

M denote the modulation order which is 2; while $Q(\cdot)$ is the standard Q function is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt \tag{8}$$

Analytical model for FFT-OFDM is studied as expressed in equation (7). In the following section these results are compared with the results of simulation of the system.

VII. SIMULATION PARAMETERS

The number of subcarriers used in OFDM system is 256, and the length of the zero-padding guard interval is 64. The system simulated does not use any channel estimation technique and error estimation or correction capabilities. BPSK modulation scheme is used.

All of the simulations assume that, channel-state information will not change in one OFDM symbol. To ensure the reliability of the computer simulations, OFDM symbols are generated to obtain each BER value in the simulations. BER performance for different wavelet mothers namely Harr, Daubechie (dB2), Symlets (sym2), Coeiflet (coifi2) and Bi-orthogonal is compared. The simulation is carried out for AWGN channel.

VIII. RESULTS

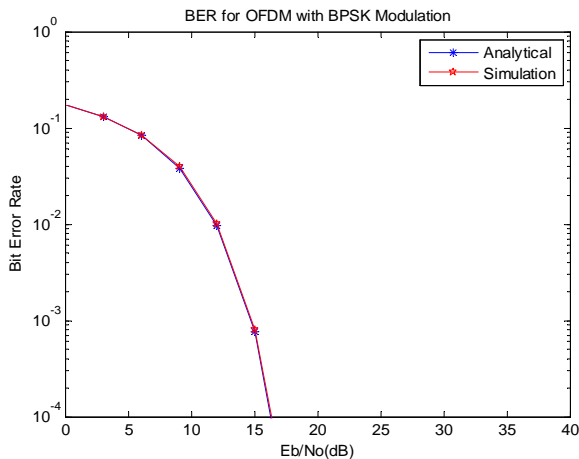


Fig.3. BER Performance of analysis and simulation of OFDM

Simulation model indicated in Fig.1. is used; BPSK modulation is used with a suitable cyclic prefix. Simulation is carried out for E_b/N_0 in the range 0 to 40dB and for BPSK as a modulation technique. Fig.3. compares a conventional OFDM system BER performance of an analytical model and simulation model.

Results indicate that BER performance of analytical model and simulation model are consistent. Hence we can comment that the simulation model designed for FFT-OFDM is validated.

The wavelet based OFDM system is simulated for various mother wavelets. Simulation model is similar to Conventional OFDM, only cyclic prefix is not needed for DWT-OFDM system. DWT-OFDM system is shown in Fig.2. Simulation is carried for different mother wavelets namely Haar, Daubechies, Biorthogonal, Symlet, coeiflet

and for E_b/N_0 in the range of 0 to 40 dB. Fig.4 shows comparative results of the simulation of FFT-OFDM and DWT-OFDM for aforementioned mother wavelets. In this simulation, we used cyclic prefix of 25% of the total OFDM symbol period for the FFT-OFDM system. The DWT-OFDM families do not require cyclic prefix due to the overlapping nature of their properties. Results shows wavelet OFDM outperforms conventional OFDM for lower signal power or E_b/N_0 value less than 25% Mother wavelets Haar and Biorthogonal perform better than other wavelets for all values of E_b/N_0 . After E_b/N_0 more than 10 dB dB2, symlet and coeiflet do not demonstrate further improvement in performance. These mother wavelets (sym2, coifi1 and dB2) BER remain almost constant at non-acceptable level of 2×10^{-2} . In general for a communication system the BER should be better than 10^{-3} ; the simulations shown that for an E_b/N_0 of 10 dB or more

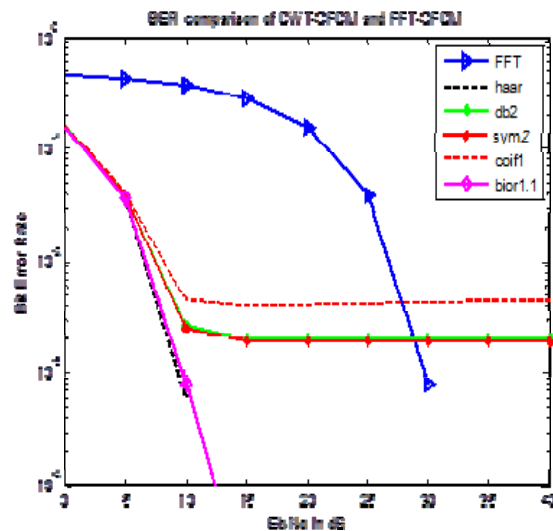


Fig.4. Performance comparison of DWT-OFDM and FFT-OFDM

BER is higher than 10^{-3} for Haar and Biorthogonal mother wavelets. Fig.5 illustrates the BER performance of Wavelet-OFDM system by using different orders of biorthogonal mother wavelets. Here bior 1.1 and bior 3.1 shows satisfactory bit error rate up to 10^{-4} for 10 dB SNR. So after performing this experiment, we can recommend that biorthogonal mother wavelet gives better performance parameters for implementing DWT-OFDM.

IX. CONCLUSION

From the analysis and simulation we found that the designed simulation model is trustworthy. For DWT-OFDM haar and bior1.1 mother wavelets will be the best choice for implementation.

The next objective of this work is to design and implement a base band OFDM transmitter and receiver on FPGA hardware. This project concentrates on developing discrete wavelet transforms (DWT) and Inverse Discrete wavelet transform (IDWT). The work also includes in designing a

mapping module, serial to parallel and parallel to serial converter module. For FPGA implementation first complete Simulink model of FFT-OFDM and DWT-OFDM have to be tested. Based on these results for FPGA implementation Haar wavelet is recommended. By using HDL code generation or by using appropriate Xilinx block sets we have to implement this complete model into Spartan-3 environment.

All modules will be designed using VHDL programming language and implement using Spartan-3 board. The board is connected to computer through parallel port and development kit is used to provide interface between user and the hardware. All processing is executed on Spartan-3 board and user only requires giving the inputs data to the hardware throughout. Input and output data is displayed to computer and the results is compared using MATLAB software. Software's which are used in this project includes VHDL, Xilinx 13.1. Software tools will be used to assist the design process and downloading process into FPGA board.

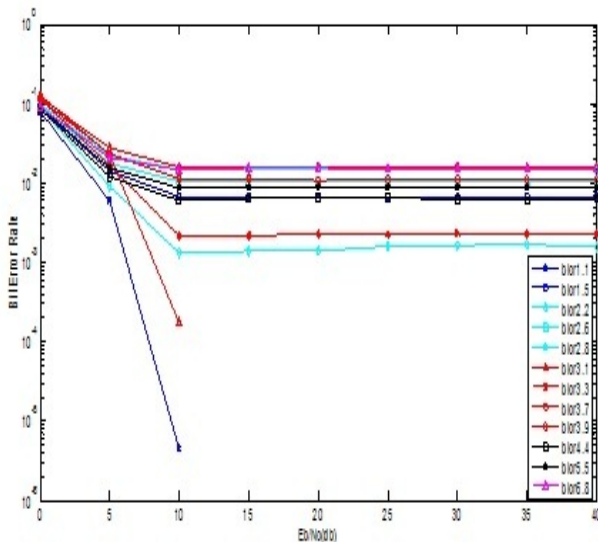


Fig.5.BER performance of DWT-OFDM using biorthogonal mother wavelets

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